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Non-Manipulable Mechanisms: A Brief Overview

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Summary

Manipulation of a mechanism by the principal himself is an issue in a context where there is no grand-mechanism linking the principal and his agents who are instead tied together by a set of bilateral contracts. Taking into account the possibility of manipulations and focusing on mechanisms which are robust to such manipulations restores continuity of the optimal mechanism with respect to the information structure and goes in the direction of modelling weaker institutions than currently assumed in the mechanism design literature. We present an overview of our ongoing research on the subject and highlight some promising applications.

Résumé

Nous étudions la manipulation des mécanismes incitatifs par le principal lui-même dans un contexte où, d'une part, il n'est pas possible de signer un grand contrat liant ce principal à l'ensemble de ses agents simultanément et, d'autre part, seuls des contrats bilatéraux sont possibles. Prendre en compte cette possibilité de manipulations et se concentrer sur les mécanismes robustes à de telles manipulations permet de rétablir la continuité des mécanismes optimaux par

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rapport à la structure d'information. Cette approche contribue aussi au développement de la modélisation d'institutions plus faibles que celles retenant généralement l'attention dans la littérature. Nous présentons un survol de quelques-uns des résultats de notre programme de recherche sur ce thème.

Keywords: Mechanism Design, Manipulation, Bilateral contracting.

Mots clés : Mécanismes incitatifs, manipulations, contrats bilatéraux.

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1. Introduction

Earlier on Samuelson (1954) showed that Pareto optimal allocations could be decentralized in a public good context by means of personalized Lindahl prices. It was soon recognized even by Samuelson himself that the agents' private information on their preferences for the public good would be a major impediment to this market solution to the public good problem. Lindahl prices could be manipulated by privately informed agents adopting a strategic behavior. Following the seminal works of Clarke (1971), Groves (1973) and Green and Laffont (1979), economic theory turned thus to the design of more complex incentive compatible mechanisms which would be robust to manipulations of preferences by privately informed agents. Through such mechanisms, privately informed agents communicate the intensity of their preferences to an uninformed planner who, in response, specifies what should be the agents' payments and the amount of public good produced. Of course, the basic tenet of this literature has been to assume that the government who proposes and implements the mechanism is endowed with some strong commitment power since he is able to bring to the contracting table all his agents within a grand-mechanism to which all abide. The Theory of Mechanism Design was born.

This theory is traditionally concerned with the problem of manipulation of the choice rule by different parties involved in a contract. The manipulation opportunities have basically two sources. First, parties which are initially endowed with some private information may use it strategically to distort the outcome to their benefit. Second, parties can use for the same purpose the information they have learned in the course of the mechanism.

Much of the mechanism design literature abstracts from the second possibility by making strong assumptions about the commitment power of the principal and the public nature of communication. Nevertheless, a tiny literature studies the second possibility by focusing on *posterior implementability* (see Green and Laffont, 1987) or *ex post* implementation (see Dasgupta and Maskin, 2000 ; Bergemann and Morris, 2005, and Jehiel *et al.*, 2006). The underlying idea in these papers is that a mechanism should induce strategies that are best-responses even after agents have learned information in the course of playing the mechanism.

In our study of non-manipulable mechanisms starting with Dequiedt and Martimort (2007a), we are also concerned by the two sources of manipulation opportunities but open the second for the principal only. To do so, we depart from standard mechanism design by assuming that there is no grand-mechanism linking the principal with his agents. Instead, the principal is linked with his agents through a set of bilateral contracts. What an agent privately communicates to the principal in any such relationship can be used then by the principal himself to manipulate other relationships. In that sense, our theory goes in the direction of modelling mechanism design issues in a star network as network theorists would say (see Jackson, 2005).

In much bilateral contracting settings with an uninformed principal, there is some scope to use the information privately held by a given agent to affect the contracts of others. This may be the case because there exist informational externalities between agents (their types are correlated for instance) or production externalities (like in a team context or in an auction environment). Each bilateral contract between the principal and each of his agent should thus allow for the possibility to use what the principal has learned from other agents to determine the final allocation. With bilateral contracting, each agent is unable to check whether his own report to the principal fits with what the principal himself reports to other agents. This opens the possibility of opportunistic behavior by the principal himself. Taking into account the possibility of that opportunistic behavior of the principal imposes additional *non-manipulability* constraints on the allocations that can be implemented.

Introducing those constraints buys us two important results. First, it simplifies significantly mechanisms and optimal institutions. In various environments with different degrees of complexity, simple nonlinear prices, the so-called “sell-out” contracts (or some generalizations of them) are actually optimal. Intuitively, when the principal’s opportunism is an issue, a prior commitment to a contract which makes the principal indifferent between the various choices he may leave *ex post* to an agent helps since it is robust to any *ex post* manipulation that the principal could entertain. This does not mean that contracts are silent on most contingencies because these non-manipulability constraints still leave ample room for screening private information held by agents on the periphery of the organization. From

an *ex ante* viewpoint, the principal is of course not indifferent between all possible outputs and output distortions can be used to extract the agents' information rents.

Second, introducing the non-manipulability of the mechanism answers what can be seen as the most important puzzle of Incentive Theory: The fact that information, when (even slightly) correlated between agents of the organization, can be extracted at no cost by the principal (Cr mer and McLean, 1988 among others) whereas, in the case of independent types, optimal mechanisms reach a genuine trade-off between rent extraction and efficiency (Myerson, 1981) which is much in lines with the intuition developed for the principal-single agent model (see Laffont and Martimort, 2002). Taking into account the non-manipulability of the mechanism restores continuity with respect to the information structure for the players' payoffs but also for the optimal mechanism which implements the second-best outcome. Still, this assumption maintains correlation as a means to better (but not fully) extract the agents' information rent. This gives a more realistic picture of mechanism design in multi-agents environments.

Section 2 characterizes the set of non-manipulable mechanisms by means of both a Revelation and a Taxation Principles. Section 3 shows how this characterization is useful in contexts where agents work on independent projects for the principal and where the only externality between them is an informational one. Section 4 surveys the results obtained when applying the concept of non-manipulability to auctions. Section 5 explores public good settings. Section 6 concludes.

2. Non-Manipulability

The first step of the analysis of non-manipulable mechanisms consists in studying the perfect Bayesian equilibria of a multi-stage game where a principal offers a mechanism to the agents of his organization (actually a set of bilateral contracts), exchanges private messages with them and then chooses to publicize the report from one agent to others. To characterize such allocations, we need to rely on some version of the Revelation Principle which adds *non-manipulability constraints* on the principal's side to the usual characterization of the set of incentive feasible allocations.

In private value settings, i.e., when an agent's information (characterized by a preference or technological parameter θ_i) does not directly enter in the utility function of the principal, truthful revelation of agents' private information does not directly interact with manipulations by the principal. In Dequiedt and Martimort (2007a), we adapt the standard revelation principle to show:

Proposition 2.1 – The Revelation Principle with Bilateral Contracting.

Dequiedt and Martimort (2007a): Any allocation achieved at a continuation Perfect Bayesian Equilibrium of any arbitrary mechanism with bilateral contracting can also be implemented as a truthful and non-manipulable continuation PBE of a direct mechanism.

Therefore considering private communication within each bilateral contract linking an agent with the principal simply adds a set of non-manipulability constraints on top of the usual Bayesian incentive compatibility and participation constraints. These additional constraints completely describe how manipulation opportunities by the principal restricts the set of implementable choice rules.

The possibility of manipulating the reported messages has a counterpart in terms of manipulation of the physical variables available for contracting. To get a better understanding of non-manipulability requirements, it is useful to reformulate the problem in terms of a *modified common agency* game. Such a game highlights the bilateral nature of contracting and the extent of the principal's opportunism in such an environment.

Let us describe the timing of that game. At date $t = 1$ the principal offers a menu of nonlinear prices $\{T_i(q_i, \theta_i)\}$ to each agent of his organization. At date $t = 2$, each agent picks one nonlinear price within the menu and reveals thereby his type to the principal. Finally, at date $t = 3$ the principal chooses the vector of quantities to be implemented once informed about the agents' types by observing their reports. An equilibrium of such a game is truthful if a $\hat{\theta}_i$ -agent chooses the nonlinear price $T_i(q_i, \hat{\theta}_i)$ corresponding to his type at date $t = 2$. Still in the case of private values, we can show:

Proposition 2.2 – The Taxation Principle.

Dequiedt and Martimort (2007a): Any allocation achieved at a continuation Perfect Bayesian Equilibrium of any arbitrary mechanism with private communication can also be implemented as a truthful continuation PBE of a modified common agency game.

This formulation highlights the scope for the principal's opportunism. Here the principal cannot commit *ex ante* to a given quantity schedule but only to nonlinear prices and will choose how much activities he requests from each agent *ex post*. However, the departure from standard mechanism design theory is rather minimal and only half-way that made in the common agency models. Contrary to those common agency models, we let here the party at the node of all the contracts play first. This extra step allows to replace the complexity of looking at complex Nash equilibria in contracts by the more natural and simple tools of optimization. This simplifies significantly the analysis and gets rid of the multiplicity problem that arises in those common agency games still capturing much of the economic

content from assuming opportunistic behavior of the party at the nexus of all contracts.

Let us quickly provide two examples which give some flavor of the research program opened when considering the non-manipulability of a mechanism.

Example 1: Vickrey auction: The Taxation Principle can be used to get a first insight on non-manipulability. Consider the allocation of a single unit of an indivisible private good. The Vickrey auction allocates the good to the agent with the highest valuation and implements a monetary transfer equals to the second-highest valuation. It is straightforward to see that in this case, the Vickrey mechanism cannot be expressed in terms of a *modified common agency* game because agents do not pay according to a nonlinear price schedule. What an agent pays for the good is not a constant but depends on what the other agents claim. Therefore, we can immediately conclude that the Vickrey auction is manipulable by the principal. Trivially, a principal learning the report of an agent on his preferences through a bilateral relationship may want to say that the losing bid was just below to optimize the winner's payment. A contrario, the first-price auction is non-manipulable although it is generally not optimal beyond the case of independent types.

Example 2: Myerson's optimal auction: The impact of non-manipulability is important in cases private information is correlated. Consider for instance the optimal auction studied in Myerson (1981), Section 7. In Myerson's example, two bidders have private valuations in $\{10, 100\}$ with $Pr(10, 10) = Pr(100, 100) = \frac{1}{3}$, and $Pr(100, 10) = Pr(10, 100) = \frac{1}{6}$. With full commitment of the seller, it is possible to design an auction that extracts all the surplus from the bidders in a Bayes-Nash equilibrium by implementing the transfers t (from the bidders to the seller) and allocation rule q defined by

$$\begin{cases} t(100, 100) = (50, 50), & q(100, 100) = (\frac{1}{2}, \frac{1}{2}), \\ t(100, 10) = (100, 30), & q(100, 10) = (1, 0), \\ t(10, 100) = (30, 100), & q(10, 100) = (0, 1), \\ t(10, 10) = (-10, -10), & q(10, 10) = (\frac{1}{2}, \frac{1}{2}). \end{cases}$$

This auction is easily manipulable by the principal if the agents are linked through bilateral contracts and de facto do not observe each other bids. Indeed, after a bidder has announced 10, it is in the seller's interest to pretend that the other bidder has announced 100 and reap the corresponding payment.

3. Separable Projects

Let us briefly describe our set-up. We consider an organization made of one principal (P) and 2 agents (A_i for $i = 1, 2$). Agent A_i produces a good in quantity

q_i on the principal's behalf. The vector of goods (resp. transfers) is denoted by $q = (q_1, q_2)$ (resp. $t = (t_1, t_2)$). Players have quasi-linear utility functions defined respectively as:

$$V(q, t) = \sum_{i=1}^n S(q_i) - t_i \quad \text{and} \quad U_i(q, t) = t_i - \theta_i q_i.$$

The principal's surplus function $\tilde{S}(\cdot)$ is separable:

$$\tilde{S}(q_1, q_2) = \sum_{i=1}^2 S(q_i).$$

for some $S(\cdot)$ increasing and concave in q_i . The vector of goods q (resp. transfers t) belongs to some set $Q = \Pi_{i=1}^2 Q_i \subset \mathbb{R}_+^2$ (resp. $\mathcal{T} = \Pi_{i=1}^2 \mathcal{T}_i \subset \mathbb{R}^2$).

The efficiency parameter θ_i is A_i 's private information. It belongs to a set $\Theta = [\underline{\theta}, \bar{\theta}]$. A vector of types is denoted $\theta = (\theta_1, \theta_2)$. Types are jointly drawn from the common knowledge non-negative and atomless density function $\tilde{f}(\theta)$ whose support is Θ^n . For future reference, we will also denote the marginal density and the corresponding cumulative distribution respectively as:

$$f(\theta_i) = \int_{\Theta} \tilde{f}(\theta_i, \theta_{-i}) d\theta_{-i} \quad \text{and} \quad F(\theta_i) = \int_{\underline{\theta}}^{\theta_i} f(\theta_i) d\theta_i.$$

A direct revelation mechanism is of the form $\{t_i(\theta_i, \theta_{-i})\}_{i=1,2}$. The implementation concept between the agents may be Bayesian or dominant strategy as specified at the outset by the modeler. In this section, we will focus on the case of Bayesian implementation.

Written in terms of direct mechanisms, the non-manipulability constraint tells us that the principal's payoff from contracting with an agent should not depend on what he may have learned from the other through private communication. This gives the existence of an arbitrary function $h_i(\theta_i)$ such that:

$$S(q_i(\theta_i, \theta_{-i})) - t_i(\theta_i, \theta_{-i}) = h_i(\theta_i) \quad (1)$$

Equation (1) shows that each agent is made residual claimant for the part of the principal's objective function which is directly related to his own output. The nonlinear price which achieves this objective is a *sell-out contract*:

$$T_i(q_i, \theta_i) = S(q_i) - h_i(\theta_i). \quad (2)$$

Everything happens thus as if agent A_i had to pay upfront an amount $h_i(\theta_i)$ to produce on the principal's behalf. Then, the agent enjoys all returns $S(q)$ on the project he is running for the principal. The principal's payoff in his relationship with A_i is $h_i(\theta_i)$ and this payoff does not depend on the amount produced. Of course, fixed-fees are adapted so that participation by all types is ensured.

Such mechanisms form a strict subset of the set of incentive feasible mechanisms which puts much structure on optimal contracting. As a result the full extraction of the agents' information rent is no longer feasible in correlated environments and we get:

Proposition 3.1 – Dequiedt and Martimort (2007a): Assume that the generalized virtual marginal costs defined as

$$\varphi(\theta_i, \theta_{-i}) = \theta_i + \frac{\frac{F(\theta_i)}{f(\theta_i)}}{1 + \frac{\tilde{f}_{\theta_i}(\theta_{-i}|\theta_i)}{\tilde{f}(\theta_{-i}|\theta_i)} \cdot \frac{F(\theta_i)}{f(\theta_i)}}$$

are always non-negative, strictly increasing in θ_i and decreasing in θ_{-i} and that the correlation is weak enough. Then, the optimal non-manipulable Bayesian mechanism entails a downward output distortion $q^{SB}(\theta_i, \theta_{-i})$ which satisfies the following “modified Baron-Myerson” formula

$$S'(q^{SB}(\theta)) = \varphi(\theta_i, \theta_{-i}), \quad (3)$$

and agents always get a positive rent except for the least efficient ones.

With independent types, the principal finds useless the report of one agent to better design the other's incentives. He must give up some information rent to induce information revelation anyway. Outputs are accordingly distorted downward to reduce those rents and the standard Baron-Myerson distortions follow:

$$S'(q^{SB}(\theta)) = \theta_i + \frac{F(\theta_i)}{f(\theta_i)}. \quad (4)$$

The important point to notice is that the optimal multilateral contract with unrelated projects and independent types can be implemented with a pair of bilateral contracts which are *de facto* non-manipulable by the principal. The non-manipulability constraints have no bite in this case.

When types are instead correlated, the agents' rent can be (almost) fully extracted in this context with a continuum of types and the first-best output can be implemented at no cost if communication is public. Those schemes are manipulable and no longer used with private communication.

To understand the nature of the output distortions and the role of the correlation, it is useful to compare the solution found in (3) with the optimal mechanism had the principal contracted separately with each agent. By using correlation, the optimal contract reduces the agent's information rent. When the agents' types are positively correlated, learning from agent A_1 that his cost parameter is high enough is relatively good news on agent A_2 since it indicates that the latter is unlikely to have cheated on his type. There is less need to distort A_2 's output for rent extraction reasons and the output given by (3) is above the Baron-Myerson solution (4). The reverse happens in case of bad news.

Opportunism on the principal's side restore a genuine trade-off between rent extraction and efficiency. As a matter of fact, the optimal non-manipulable mechanism corresponds to a set of bilateral sell-out contract whatever the correlation structures between the agents' types.

Proposition 3.2 – *Dequiedt and Martimort (2007a): The optimal non-manipulable Bayesian mechanism is implemented through sell-out contracts whatever the level of correlation between the agents' types. Payoffs of the principal and the agents are continuous in the level of this correlation.*

4. Auctions

As suggested by Examples 1 and 2 above, the study of non-manipulable auctions offers an important application of our theory. On the theoretical hand, non-manipulability restores continuity of the optimal Bayesian incentive compatible auction with respect to the information structure and therefore evacuates the major cause of scepticism by auction theorists about the mechanism design methodology (see Milgrom, 2004, for instance). On the practical hand, assuming bilateral contracting and private communication between the seller and each of his bidders is certainly a convenient hypothesis to study auctions organized on the internet. The *modified common agency* formulation gives an active role to the seller and is also a consistent way to introduce shill bidding phenomena in a mechanism design framework.

In Dequiedt and Martimort (2007b), we provide a first attempt to analyze non-manipulable auctions. The setting is now characterized by utility functions defined for the principal and the agents respectively as:

$$V(q, t) = \sum_{i=1}^n t_i \quad \text{and} \quad U_i(q, t) = \theta_i q_i - t_i.$$

Even if Example 1 suggested that imposing non-manipulability may have a strong impact on what can be achieved by the principal, we first obtain an equivalence result for the case of independently distributed private information.

Proposition 4.1 – *Dequiedt and Martimort (2007b): Non-manipulability constraints have no impact on the profit that can be obtained by the seller when agents' private information is independently distributed.*

Indeed, the first-price auction is easily seen to be non-manipulable by the principal and achieves (from the Revenue Equivalence Theorem) the highest possible profit for the seller.

However, as soon as private information is correlated, non-manipulability restricts the profit that can be achieved. By the same token, the first price auction is no longer optimal.

Compared to the case of unrelated projects, the analysis is made more complex here because the non-manipulability constraints are not easily separable in the agents' identity. Therefore some preliminary steps are necessary before it is possible to solve the principal's maximization problem. Provided the agents' valuations are affiliated and the implemented quantity schedule obtained by one agent is decreasing in the valuation of the other agents we can prove that the expected rent left to the agents is minimal when the mechanism offered by the principal is an all-pay auction with a type dependent entry-fee. With such an auction, the agents pay an entry $T_i(q_i, \theta_i) = h(\theta_i)$ independent on whether he wins or not the auction; the winning agent being the one who pays the highest entry fee.

Once this step is performed, we can separate again the non-manipulability constraints as in equation (1) and the analysis of the rent-efficiency trade-off is much similar to that in the independent projects case. It is possible to derive the *generalized virtual valuations* that drive the determination of the optimal allocation. Correlation implies that even if we start from a purely private value setting in which each agent's valuation depends on his own private information only, the *generalized virtual valuations* are interdependent, they take into account an indirect informational externality. One consequence of this feature is that it is generally impossible for the principal to implement the optimal auction with a deterministic reserve price. Instead, the decision of the seller to allocate the good to the winning bidder or to keep it is driven by the whole array of bids. The reserve price is therefore stochastic as it depends on the bids of others. Deriving the optimal auction, we can prove:

Proposition 4.2 – Dequiedt and Martimort (2007b): *The optimal non-manipulable auction can be implemented via an all-pay auction with secret (and stochastic) reserve price.*

5. Public Good Mechanisms

Let us consider the scope for manipulability of public good mechanisms. This new assumption aims at modelling weaker institutions for the provision of public goods than what has generally been assumed. In particular, it relies on the fact that governments have a limited commitment ability, an issue of much practical importance in a public good context but which has received little attention so far.

Whereas in the case of auctions investigated in Section 4, there is little ambiguity in the objective of the seller, things are somewhat different for public good

provision. Indeed, the behavior of the government acting as the principal designing the mechanism depends on his objective function. Planners may be benevolent, may have redistributive concerns either postulated at the outset¹ or may act as Leviathan. In each case, the non-manipulability of the mechanism puts different kinds of constraints on the principal.

Indeed, the non-manipulability constraints take different forms depending on the exact nature of the government's objective. A benevolent planner chooses output *ex post* to maximize the sum of the agents' utilities. A Leviathan would instead act opportunistically to maximize the profits he may withdraw from producing the public good on the agents' behalf.

To fix ideas, consider a two-agent economy. Agent A_i has a constant marginal valuation for the public good which is denoted by θ_i . Incorporating the cost of the public good into the agents' utility functions, agent A_i 's utility function can be written as:

$$U_i = v(\theta_i, q) - t_i \text{ where } v(\theta_i, q) = \theta_i q - \frac{q^2}{2}.$$

In such environment, it is straightforward to check that the first-best output satisfying the Lindahl-Samuelson condition is:

$$q^*(\theta_1, \theta_2) = \frac{1}{2}(\theta_1 + \theta_2).$$

Consider now a dominant strategy (or Clarke-Groves) mechanism $\{t_i(\theta_i, \theta_{-i})\}_{i=1,2}$. Such mechanism satisfies the following dominant strategy incentive compatibility:

$$\frac{\partial t_i}{\partial \theta_i}(\theta_i, \theta_{-i}) = (\theta_i - q^*(\theta_i, \theta_{-i})) \frac{\partial q_i^*}{\partial \theta_i}(\theta_i, \theta_{-i}).$$

Therefore, we get by integration:

$$t_i(\theta_i, \theta_{-i}) = \frac{\theta_i^2}{8} - \frac{\theta_i \theta_{-i}}{4} - g(\theta_{-i})$$

for some $g(\cdot)$ function. In terms of nonlinear prices, and using $\theta_{-i} = 2q - \theta_i$ to replace into the previous formula, these transfers can be written as:

$$T_i(\theta_i, q) = \frac{3\theta_i^2}{8} - \frac{\theta_i q}{2} - g(2q - \theta_i).$$

Consider now the problem of a benevolent but opportunistic principal. First, this principal offers such menus to the agents and let them pick a particular item in those menus. From the dominant strategy requirement, this choice is incentive compatible:

$$\theta_i \in \arg \max_{\hat{\theta}_i} v(\theta_i, q^*(\hat{\theta}_i, \theta_{-i})) - T_i(\theta_i, q^*(\hat{\theta}_i, \theta_{-i})).$$

1. Ledyard and Palfrey (1999).

Once informed on the agents' preferences by this first stage of the game, the principal chooses *ex post* the amount of public good $q(\theta_1, \theta_2)$ so that:

$$q(\theta_1, \theta_2) = \arg \max_q \sum_{i=1}^2 v(\theta_i, q) - T_i(\theta_i, q).$$

Restricting to $g(\cdot)$ differentiable and concave in q , this yields the following first-order condition:

$$q^*(\theta_1, \theta_2) - 2q(\theta_1, \theta_2) = 2 \left(\sum_{i=1}^2 g'(2q(\theta_1, \theta_2) - \theta_i) \right). \quad (5)$$

The Clarke-Groves mechanism is non-manipulable by the principal if and only if $q^*(\theta_1, \theta_2) = q(\theta_1, \theta_2)$ solves (5). This yields that necessarily

$$\sum_{i=1}^2 g'(\theta_i) = - \sum_{i=1}^2 \frac{\theta_i}{4}$$

and

$$g(\theta_i) = -\frac{\theta_i^2}{8} + k$$

where k is some constant. We can thus state:

Proposition 5.1 – *Up to some constant, there exists a unique Clarke-Groves mechanism which is non-manipulable.*

This result might be contrasted with example 1 which deals with a single unit auction and establishes that the Vickrey auction is manipulable. In a different context, non-manipulable Clarke-Groves mechanisms might still exist.

6. Conclusion

Non-manipulability can be interpreted as incentive compatibility on the principal's side. It is a notion that is consistent with relaxing the commitment hypothesis made in standard mechanism design theory and is justified by a simple hypothesis of bilateral contracting and private communication between the principal and each of his agents. Beyond the fact that non-manipulability constraints restore continuity of the optimal Bayesian mechanism with respect to the information structure, it is possible to reconsider the entire theory of incentives by adding such constraints to obtain more satisfactory mechanisms closer to real-world institutions. We believe that such a reconsidered theory can be useful to understand a wide range of phenomena like auctions on the internet, the provision of local public

goods or intermediated trading institutions. Much work remains to be done and we hope this overview will become rapidly obsolete.

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